

# On the concept of effective temperature in current carrying quantum critical states

Stefan Kirchner <sup>\*1</sup>, Qimiao Si <sup>2</sup>

<sup>1</sup> Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Str. 38, D-01187 Dresden, Germany

<sup>2</sup> Department of Physics & Astronomy, Rice University, Houston, TX, 77005, USA

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\* Corresponding author: e-mail kirchner@pks.mpg.de, Phone: +49-351-871-1121

Quantum criticality has attracted considerable attention both theoretically and experimentally as a way to describe part of the phase diagram of strongly correlated systems. A scale-invariant fluctuation spectrum at a quantum critical point (QCP) implies the absence of any intrinsic scale. Any experimental probe may therefore create an out-of-equilibrium setting; the system would be in a non-linear response regime, which violates the fluctuation-dissipation theorem (FDT).

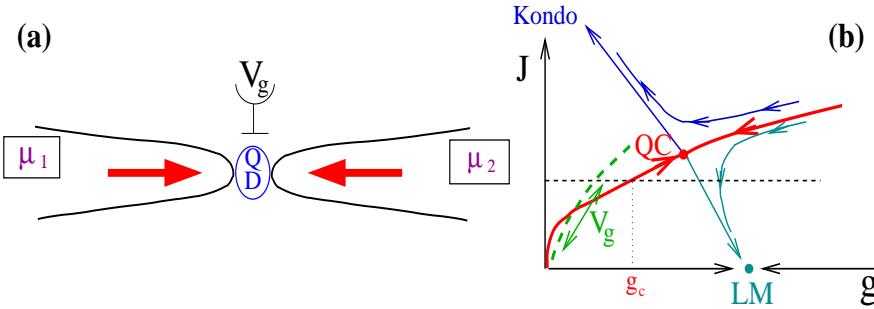
Here, we study this violation and related out-of-equilibrium phenomena in a single electron transistor (SET) with ferromagnetic leads, which can be tuned through a quantum phase transition. We review the breakdown of the FDT and study the universal behavior of the fluctuation dissipation relation of various correlators in the quantum critical regime. In particular, we explore the concept of effective temperature as a means to extend the FDT into the non-linear regime.

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**1 Introduction** Quantum criticality has become a new paradigm in describing complex quantum matter. It occurs when matter goes continuously from one phase to another at zero temperature as a function of a control parameter. No classification scheme into universality classes has emerged so far, but it has become clear that not all QCPs are characterized by order parameter fluctuations alone. Rather, inherently quantum modes may become critical and must also be incorporated in the description. Such a QCP is not described by a Ginzburg-Landau-Wilson functional and has no classical (finite temperature) counterpart. The strongest evidence to date for the existence of such unconventional quantum criticality has come from heavy fermion systems at the border to magnetism. One of the potential mechanisms for quantum-critical heavy fermion metals, that is beyond the Ginzburg-Landau-Wilson paradigm, is the critical destruction of the Kondo effect [1, 2, 3, 4]. This destruction is local in space and leads to interacting critical modes along imaginary time. Out-of-equilibrium states near quantum phase transitions, however, have so far received only limited theoretical

attention despite a long-standing strong interest in their classical counterparts. Experimentally, on the other hand, it seems unavoidable to generate out-of-equilibrium states during a measurement in the quantum critical regime. This regime is characterized by the absence of any intrinsic scale. Any measurement may therefore potentially perturb the system beyond the linear response regime where the FDT no longer holds<sup>1</sup>. Recently, we demonstrated that a local quantum critical point can be realized in a SET attached to ferromagnetic leads [5]: As the applied gate voltage is varied, the magnetic SET can be tuned through a continuous quantum phase transition where the Kondo effect is critically destroyed [5, 6]. Such a nano device also constitutes a simplified system, both theoretically and experimentally, to study well-defined out-of-equilibrium states that give rise to unique steady-state limits. In order to elaborate on this insight, we identified a limit where both

<sup>1</sup> The FDT states that, for a small enough external drive, the system will react in a way prescribed by its equilibrium fluctuations.



in- and out-of equilibrium properties of the magnetic SET can be determined exactly [7]. We studied the non-linear current-voltage (or I-V) characteristics for current-carrying steady states across the quantum transition [7].

In the present work, we elaborate on the fate of the fluctuation-dissipation relation in the non-linear regime and explore the notion of effective temperature.

## 2 The quantum critical single-electron transistor

The magnetic SET, a quantum dot attached to ferromagnetic leads, has been investigated experimentally and theoretically [8, 9, 10, 5, 6]. The couplings of the local degrees of freedom to the conduction electrons and to the magnons in the leads allow for a dynamical competition between the Kondo singlet formation and the magnon drag. As a result, the low-energy properties are governed by a Bose-Fermi Kondo model (BFKM) [5, 6].

$$\mathcal{H}_{\text{bfk}} = \sum_{i,j} J_{i,j} \mathbf{S} \cdot c_{k,\sigma,i}^\dagger \frac{\sigma}{2} c_{\sigma',j} \quad (1)$$

$$+ \sum_{\mathbf{k}, i, \sigma} \tilde{\epsilon}_{\mathbf{k}\sigma i} c_{\mathbf{k}\sigma i}^\dagger c_{\mathbf{k}\sigma i} + h_{\text{loc}} S_z \quad (2)$$

$$+ g \sum_{\beta, \mathbf{q}, i} S_\beta (\phi_{\beta, \mathbf{q}, i} + \phi_{\beta, \mathbf{q}, i}^\dagger) + \sum_{\beta, \mathbf{q}, i} \omega_{\mathbf{q}} \phi_{\beta, \mathbf{q}, i}^\dagger \phi_{\beta, \mathbf{q}, i}$$

where  $i, j \in \{L, R\}$  and  $h_{\text{loc}} = g \sum_i m_i$ , is a local magnetic field with  $m_L/m_R$  being the ordered moment of the left/right leads. For antiparallel alignment and equal couplings one finds  $m_L = -m_R$ .  $\tilde{\epsilon}_{\mathbf{k}\sigma i}$  is the Zeeman-shifted conduction electron dispersion, and  $\phi_{\beta, i}$ , with  $\beta = x, y$ , describes the magnon excitations. The spectrum of the bosonic modes is determined by the density of states of the magnons,  $\sum_q [\delta(\omega - \omega_q) - \delta(\omega + \omega_q)] \sim |\omega|^{1/2} \text{sgn}(\omega)$  up to some cutoff  $\Lambda$ . A sketch of the magnetic SET is shown in Fig. 1(a). The resulting phase diagram of the system is displayed in Fig. 1(b). For further details, see [5, 6, 7]. Nonequilibrium states are created by keeping left and right lead at different chemical potentials but the same temperature  $T$ , ( $eV = \mu_L - \mu_R$ ). The current between the dot and the right lead (R) for an arbitrary bias voltage  $V$  is [11]

$$I_R = \frac{ie}{\hbar} \int d\omega \rho_R(\omega) [f_R(\omega)(T_{RR}^r(\omega) - T_{RR}^a(\omega)) + T_{RR}^<(\omega)],$$

where  $T_{\alpha, \beta}$ , ( $\alpha, \beta = L/R$ ) is the T-matrix of the BFKM,  $f_{L/R}(\omega) = f(\omega \pm \mu_{L/R})$  is the Fermi function for the

**Figure 1** (a) Principal setup of the magnetic SET. The arrows in the left and right leads indicate the magnetization in the leads. For antiparallel alignment,  $h_{\text{loc}}$  in Eq.(3) vanishes. (b) Phase diagram: the Fermi liquid phase ('Kondo') and the critical local moment ('LM') phase are separated by a QCP. The gate voltage  $V_G$  allows to tune the system through a quantum phase transition.

left/right lead and  $T^{r/a}<$  denote the retarded, advanced and lesser T-matrix respectively. These three functions together define the "larger" function:  $T^> = T^r - T^a + T^<$  and the corresponding fluctuation-dissipation ratio (FDR) becomes

$$FDR_T \equiv \frac{T^> + T^<}{T^> - T^<}, \quad (3)$$

which, in equilibrium, reduces to  $1 - 2f(\omega) = \tanh(\omega/2T)$  (or to  $1 + 2b(\omega) = \coth(\omega/2T)$ , in the case of bosonic variables), in accordance with the FDT. Eq.(3) implies

$$T^< = \frac{1}{2}(FDR_T - 1)[T^r - T^a]. \quad (4)$$

The spin exchange matrix between the leads and the local moment arises out of virtual charge fluctuations. We therefore have  $J_{RR} = \alpha J_{LL}$ , which implies  $T_{RR} = \alpha T_{LL}$ . An equation similar to the one for  $I_R$  holds for the current between the left lead (L) and the dot,  $I_L$ . In the steady state limit, these two have to be equal:  $I_L + I_R = 0$ . (We will only consider the steady state limit here, where correlation functions depend on time differences:  $\langle A(t_j)B(t_i) \rangle = \langle A(t_j - t_i)B(0) \rangle$ .) For simplicity, we take  $\rho_R(\omega) = \rho_L(\omega) = \rho(\omega)$ . The steady state condition then implies

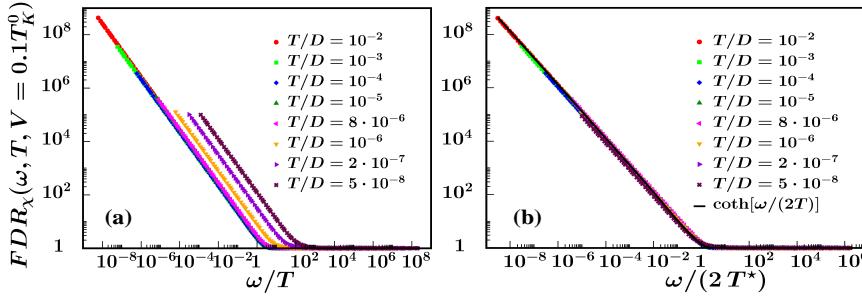
$$\int d\omega \rho(\omega) [f_L(\omega) + \alpha f_R(\omega) + \frac{1+\alpha}{2}(FDR_T - 1)] \times (T_{LL}^r(\omega) - T_{LL}^a(\omega)) = 0,$$

and since  $\rho(\omega) \geq 0$ ,  $T_{LL}^r(\omega) - T_{LL}^a(\omega) = 2\text{Im}T(\omega) \geq 0$ , once can solve for  $FDR_T$ :

$$FDR_T(\omega, T, V) = \frac{\sinh(\omega/T)}{\cosh(\omega/T) + \cosh(V/2T)}, \quad (5)$$

where we assumed equal couplings ( $\alpha = 1$ ). This equation was used in [7] to infer the scaling properties of the T-matrix in the quantum critical regime from the scaling of the  $I - V$  characteristics.

We now turn to the concept of effective temperature in the non-linear regime. The notion of an effective temperature for extending the FDT to non-equilibrium states was first introduced in the context of steady states in chaotic systems [12], and was later used for non-stationary states in glassy systems [13]. For the ohmic spin-boson model, a variant of the BFKM, no meaningful effective temperature can be defined [14]. Here, we have an exact expression for



**Figure 2** (a) The fluctuation-dissipation ratio for the spin susceptibility in the critical local moment phase ( $g = 4 \cdot g_c$ ) for  $V/D = 5.0 \cdot 10^{-3} = 0.1T_K^0$ , where  $T_K^0$  is the Kondo temperature in the absence of magnons ( $g = 0$ ). (b) An effective temperature  $T_\chi^*$  can be defined such that the  $FDR_\chi(\omega, T, V)$  of (a) collapses on  $\coth(\omega/(2T))$ , the equilibrium FDR of a bosonic field.

the FDR of the T-matrix in the steady state limit. It is easy to show that an effective temperature in the limit where  $\omega \ll T$  (and  $\alpha = 1$ ) can be obtained from Eq. (5):

$$\tanh(\omega/T^*) \equiv FDR_T(\omega, T, V) \Big|_{\omega \ll T} \\ \Rightarrow T_T^*(T, V) = \frac{1}{2}(T + T \cosh(V/2T)). \quad (6)$$

This effective temperature restores the FDT for  $T(\omega, T, V)$  everywhere in the phase diagram irrespectively of whether the system is critical or not and is ultimately connected to the boundary condition that maintains the steady state, i.e., a time-independent particle flux through the magnetic SET. For no other correlator is the associated FDR completely determined by the steady state condition. In Ref. [7], we argued that generally, FDRs in the quantum critical regime of the magnetic SET displays both,  $\omega/T$ - and  $V/T$ -scaling. In order to address whether it is possible to introduce effective temperatures in the quantum critical regime for correlators other than  $T$ , we focus in the following on the dynamical spin response. The dynamical spin susceptibility in the current-carrying steady state at arbitrary bias voltage  $V$  has been explicitly studied by an extension of the dynamical large-N of Ref. [15] onto the Keldysh contour [7]. Fig. 2(a) displays the  $FDR_\chi(\omega, T, V)$  for  $V = 5 \cdot 10^{-3}D = 0.1T_K^0$ , with  $T_K^0$  being the Kondo temperature in the absence of magnons ( $g = 0$ ). The scaling regime extends up to energies of the order of  $T_K^0$ , so that bias voltages larger than  $V = 0.1T_K^0$  might be affected by sub-leading contributions. Three important observations underly the results displayed in Fig. 2(a): (1) for  $\omega \gg T$ ,  $FDR_\chi(\omega, T, V)$  approaches the value predicted by the FDT, ( $\coth(\omega/2T) \rightarrow 1$ ), (2) for  $T > \approx 10^{-3}V$ , the deviations from the linear response behavior are hardly discernible, (3) for  $T < \approx 10^{-3}V$ , it appears as if a simple scaling factor could collapse  $FDR_\chi(\omega, T, V)$  on the high-temperature curve, where (for  $T \gg V$ ) the FDT applies. (1)-(3) suggest, that an effective temperature of the form

$$T_\chi^*(T, V) = \frac{T}{\tanh(\alpha \frac{2T}{V})} \quad (7)$$

could restore the FDT for all frequencies  $\omega$ .

As Fig. 2(b) demonstrates, this is indeed the case! The value of the scale factor between  $T$  and  $V$  is  $\alpha = 10^3$ ,

as suggested by observation (2). It is worth noting that the same  $T_\chi^*(T, V)$  applies to other  $V$  in the scaling regime (with the same  $\alpha$ ). The scaling factor  $\alpha$  is determined by some dimensionless combination of the 'non-universal' parameters  $D, g_c, T_K^0$  and  $\Lambda$ .

While both effective temperatures are formally  $\omega$ -independent,  $T_T^*$  applies only in the limit  $\omega \ll T$ , and has a singular  $T \rightarrow 0$ -limit (with  $V \neq 0$ ). By contrast,  $T_\chi^* \rightarrow V/(2\alpha)$  in the same limit.

In conclusion, we have shown that both, the (fermionic) T-matrix and the (bosonic) dynamical spin susceptibility allow a description of their respective fluctuation-dissipation ratio in terms of an effective temperature in the nonlinear, quantum critical regime (at least when the probing frequency is much less than the temperature). The effective temperature  $T_\chi^*$  is defined such that it restores the standard form of the FDT for local (i.e. on the SET) fermionic or bosonic variables in the nonequilibrium steady-state limit. The notion of temperature ( $T$ ) in the present case is well defined as it refers to the leads, which are kept at equal temperature. Whether the concept of effective temperature in the quantum critical regime, applied here to the local T-matrix and spin-spin correlator, can be extended to other correlators as well, will be explored in a forthcoming publication.

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